INVESTIGATIONS ON QUARRY STONE TOE BERM STABILITY

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Model test results from four experimental studies have been compiled in a data set with 687 test results. Three widely used or recently developed toe stability formulae have been validated against these model test results. It was found that all three formulae suffer from a lack of accuracy and general validity. This is caused by shortcomings of the underlying wave flume studies. More precisely, the wave height and the water depth above the toe berm are two main influence parameters for the toe berm stability and they are not independent in these studies. Testing the interdependence of parameters is therefore recommended for wave flume studies. An alternative toe berm stability formula was developed by a step-wise approach starting with a simple case with a minimum number of influencing factors and followed by more complex cases. The new formula is believed to provide a more meaningful description of the toe berm stability than existing formulae; this however requires further substantiation. The new approach is proposed as a working hypothesis for further studies and is recommended as benchmark for toe berm testing and design.

Keywords: rubble-mound breakwater; rock toe; toe berm stability

INTRODUCTION

The toe of most rubble mound breakwaters is protected by a quarry stone toe berm that is placed on a bedding layer. The hydraulic stability of this toe berm is commonly assessed by empirical formulae; the approach of van der Meer (1998) [vdM98] is widely used. It combines the toe stability formulae of Brebner and Donelly (1962) and of Gerding (1993). However, vdM98 tends to overestimate the stability of embedded toes and underestimates toe berms on a thick bedding layer. The formula was therefore modified by Muttray (2013) [M13]. An alternative toe stability formula that includes the size of the toe berm and a fictitious flow velocity on the toe berm was proposed recently by van Gent and van der Werf (2014) [GW14].

Existing toe stability formulae are characterised by large scatter when plotted against model test results (Muttray, 2013). This is indeed a cause of worry for designers. M13 is aiming at practical applications. The problem of the inherent uncertainties was bypassed by providing a somewhat conservative estimate of the toe stability. In contrast to all other formulae, model test are closely reproduced by GW14 indicating that this approach would be a major advancement.

The initial aim of this paper was refining M13 and extending the application range to deeper water. This objective was changed after publication of GW14 to an analysis of strength and weaknesses of existing formulae in order to provide some guidance for design. With this in mind a data base of toe stability tests has been set up covering a wide range of toe structures and test conditions. The evaluation focused on vdM98, M13 and GW14; a review of earlier formulae can be found in Muttray (2013). Significant shortcomings have been identified for all three formulae. The underlying reasons have been investigated and based on these findings an alternative approach has been developed. The various steps of this analysis are presented in this paper.

INFLUENCE PARAMETERS OF THE TOE BERM STABILITY

The stability of individual stones on a toe berm is commonly described by the stability number, $H_s/(\Delta D_{n50})$ combining significant wave height, H_s with median nominal rock diameter, D_{n50} and relative density of submerged stones, $\Delta\,(=\rho_s/\rho-1)$. The latter includes the density of rock, ρ_s and water, ρ. The stability number can be considered as a simplified description of the driving and resisting forces on an individual stone (Muttray, 2013) and is in this regard comparable to the factor of safety that is used in geotechnical engineering for slope stability analysis.

Besides wave height, rock size and relative density (H_s, D_{n50} and Δ) the submergence of the toe berm, d_t was identified by Brebner and Donelly (1962) as key parameter for the toe stability. All toe stability formulae include these four parameters. More recent formulae include further the damage number, N_{od} defined by the number of displaced stones in a toe section of width D_{n50}. It is arguable if N_{od} should be considered as a governing parameter for the toe stability. It could also be considered as a data analysis strategy that allows the inclusion of test results with little or excessive damage in the analysis.

A toe stability formula that includes the obvious parameters (D_{n50}, Δ, H_s and d_t) and the damage number, N_{od} should have the following form:

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$$\frac{H_s}{\Delta D_{n50}} = f(d_t,\ N_{od}, \ldots) \tag{1}$$

At least one more parameter of unit length has to be added to arrive at a dimensionally correct equation. Potential influence parameters of unit length are the water depth and wave length in front of the structure, d and L_p as well as the width and height of the toe berm, B_t and h_t. Other parameters that might affect the toe berm stability are the seabed slope, m (referring to a gradient of 1V:mH), the breakwater slope, n (referring to 1V:nH) and the front slope of the toe berm, n_t (referring to 1V:n_tH). These dimensionless slope parameters and the dimensions of the toe berm are defined in Fig. 1.

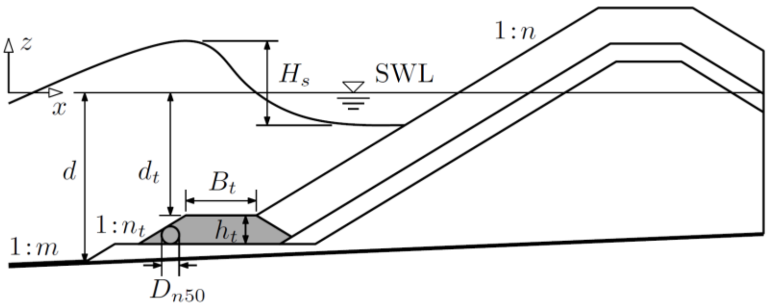

Figure 1: Definition of toe berm geometry and adjacent slopes

THE DATA SET

Model test results from wave flume experiments by Gerding (1993), Docters van Leeuwen (1996), Ebbens (2009) and van Gent and van der Werf (2014) have been compiled; the data set comprises 687 model tests. Key parameters of these tests are summarised in Table 1.

Table 1: Key parameters of wave flume tests on toe berm stability

		Gerding (1993) [G93]	Docters van Leeuwen (1996) [DvL96]	Ebbens (2009) [E09]	Van Gent & van der Werf (2014) [GW14]
Seabed slope	$1{:}m$	1:20	1:50	1:10 / 1:20 / 1:50	1:30
Breakwater slope	$1{:}n$	1:1.5	1:1.5	1:1.5	1:2
Toe width	B_t	12 / 20 / 30 cm	12 cm	10 cm	4.4 / 13.1 cm
Toe height	h_t	8 / 15 / 22 cm	8 / 15 cm	8 cm	2.9 / 5.8 cm
Rock size	D_{n50}	1.7 – 4.0 cm	1.0 – 2.3 cm	1.9 – 2.7 cm	1.5 cm
Relative density	Δ	1.68 / 2.18	0.9 / 1.55 / 1.85	1.65 – 1.75	1.7
Water depth	d	30 / 40 / 50 cm	30 / 45 cm	7.3 – 33.9 cm	20 / 30 / 40 cm
Water depth on toe	d_t	15 – 42 cm	15 – 37 cm	-0.7 – 25.9 cm	14.2 – 37.1 cm
Wave height	H_s	14.1 – 24.4 cm	9.2 – 21.1 cm	3.6 – 12.0 cm	4.8 – 24.9 cm
Wave steepness	s_p	0.01 – 0.04	0.04	0.01 – 0.04	0.015 / 0.04
Damage	N_{od}	≤ 9.2	≤ 4.7	≤ 4.4	≤ 7.3
Stability number	$\frac{H_s}{\Delta D_{n50}}$	2.1 – 8.4	2.9 – 13.9	1.1 – 4.2	1.2 – 10.5
Relative water depth on toe	d_t/H_s	0.82 – 2.78	0.88 – 3.78	-0.20 – 4.97	0.87 – 3.63
	d_t/L_p	0.06 – 0.19	0.07 – 0.19	-0.01 – 0.19	0.07 – 0.18
	d_t/d	0.45 – 0.84	0.5 – 0.82	-0.10 – 0.76	0.71 – 0.93
Number of tests results		171	98	296	122

Gerding (1993) performed toe stability tests in the Scheldt flume of Delft Hydraulics with five different toe berm geometries. The toe width varied from 12 to 30 cm (with a constant height of 15 cm) and the toe height varied from 8 to 22 cm (with constant width of 12 cm). Docters van Leeuwen (1996) conducted wave flume tests at Delft University. Worth noting about these tests is the rock material. Toe berms of basalt, porphyry and crushed brick with a specific density varying from 1,900 to 2,850 kg/m^3 were investigated. The damage was less than in other studies; it may be for this reason that these findings got little attention. Ebbens (2009) carried out model tests in the laboratory of Delta Marine Consultants (DMC) with seven different water levels. Tests with a water depth of 16 to 26 cm at the toe of the structure are similar to the conditions tested by Gerding and Docters van Leeuwen. These tests were complemented by shallow water tests with a water depth of 8 to 14 cm. Moreover, the experiments were performed with different seabed slopes. The toe berm was placed on a bedding layer of thickness 2.0 cm; in the three other studies the toe berm was placed on the seabed. Van Gent and van der Werf (2014) performed model tests in a wave flume at Deltares. Remarkable about these tests are the gentle breakwater slope and the sophisticated damage measurements. The number of displaced stones, N_{od} has been validated against independent measurements of the eroded toe profile and refers to stones that moved more than one stone diameter from their initial position. In the other studies only stones that were washed away from the toe berm had been counted.

In all four studies JONSWAP spectra were applied; the test duration was 1,000 waves except the tests of Docters van Leeuwen with 2,000 waves. A series of tests was performed with stepwise increasing wave height and all other conditions kept constant until the toe berm was severely damaged or until the limits of the wave generator were reached. The wave period was varied in these test series with the aim of having a constant wave steepness, $s_p = 2\pi H_s/(gT_p^2)$. This wave steepness may differ significantly from the actual wave steepness at the structure as it is commonly based on the deep water wave length and on the target wave height at the wave paddle. Ebbens however applied the wave length at the paddle; the wave periods in his tests are therefore somewhat longer.

Damage in the test of Gerding and Docters van Leeuwen refers to the number of displaced stones in a single test. The toe berm was repaired after each test. The cumulative damage in a series of tests was recorded by Ebbens and by van Gent and van der Werf. In these studies the toe berms were repaired when the maximum wave height in a series of tests had been reached and before continuing testing with lower wave heights.

REVIEW OF EXISTING FORMULAE

The validity of three toe toe stability formulae by van der Meer (1998), Muttray (2013) and van Gent and van der Werf (2014) has been verified against model test results. The vdM98 approach (Eq. 2) is widely used and was therefore selected. The other two formulae, M13 (Eq. 3) and GW14 (Eq. 4) are recent developments and claim to be an improvement over vdM98. All three are predictive equations for the damage number, $H_s/\,(\Delta\, D_{n50})$ being a function of damage level, toe geometry and wave conditions.

$$\frac{H_s}{\Delta D_{n50}} = \left[2 + 6.2\left(\frac{d_t}{d}\right)^{2.7}\right] N_{od}^{0.15} \tag{2}$$

$$\frac{H_s}{\Delta D_{n50}} = \frac{6\, N_{od}^{1/3}}{3.5 - \frac{d_t}{H_s}} \tag{3}$$

$$\frac{H_s}{\Delta D_{n50}} = 2.08\left[\frac{\sqrt{g}\, H_s^{0.8}\, T_p}{B_t^{0.3}\, h_t} \sinh\left(\frac{47.8}{g\, T_p^2}\, d_t\right)\right]^{1/3} N_{od}^{1/3} \tag{4}$$

Predictive equations for the damage number, N_{od} (for given rock properties, toe geometry and wave conditions) or for the required rock size, D_{n50} (for given rock density, wave conditions and toe geometry) are obtained by rearranging Eqs. 2, 3 and 4. They were validated by means of the model test results by Gerding (1993), Docters van Leeuwen (1996), Ebbens (2009) and van Gent and van der Werf (2014) (see Table 1). The stability number, rock size and damage were calculated by the three toe stability formulae (Eqs. 2, 3 and 4, rearranged if necessary) based on the reported test conditions. In other words, the reported test conditions were applied on the right hand side of the predictive equations; the predicted left hand side (calculated value) was then compared with the left hand side as observed in the experiments (measured value).

The measured and calculated values of $H_s/(\Delta D_{n50})$, D_{n50} and N_{od} are presented in Fig. 2 for vdM98, in Fig. 3 for M13 and in Fig. 4 for GW14. Different marker types are used for each of the four experimental studies. Model test results that had been used in the original derivation of the respective toe stability formula are indicated by grey-shaded markers. Transparent markers refer to other model tests results.

When comparing the stability numbers, $H_s/\Delta D_{n50}$ according to vdM98 with model test results (Fig. 2, bottom) the 90% confidence interval (CI) is given by a factor 1.5 (i.e. ranging from prediction divided by 1.5 to prediction times 1.5). This factor reduces to 1.3 when considering only the test results of Gerding (1993), which were used in the derivation of vdM98. The same can be observed for the required rock size, D_{n50} (Fig. 2, top right). The CI factor for all tests is 1.7 and reduces to 1.3 when only looking at the tests of Gerding. When predicting damage numbers, N_{od} (Fig. 2, top left) uncertainties become much larger. A CI factor of 2.0 was found for the tests of Gerding (considering only tests with $N_{od} > 0.1$). When considering all tests, there is virtually no correlation between measured and predicted damage.

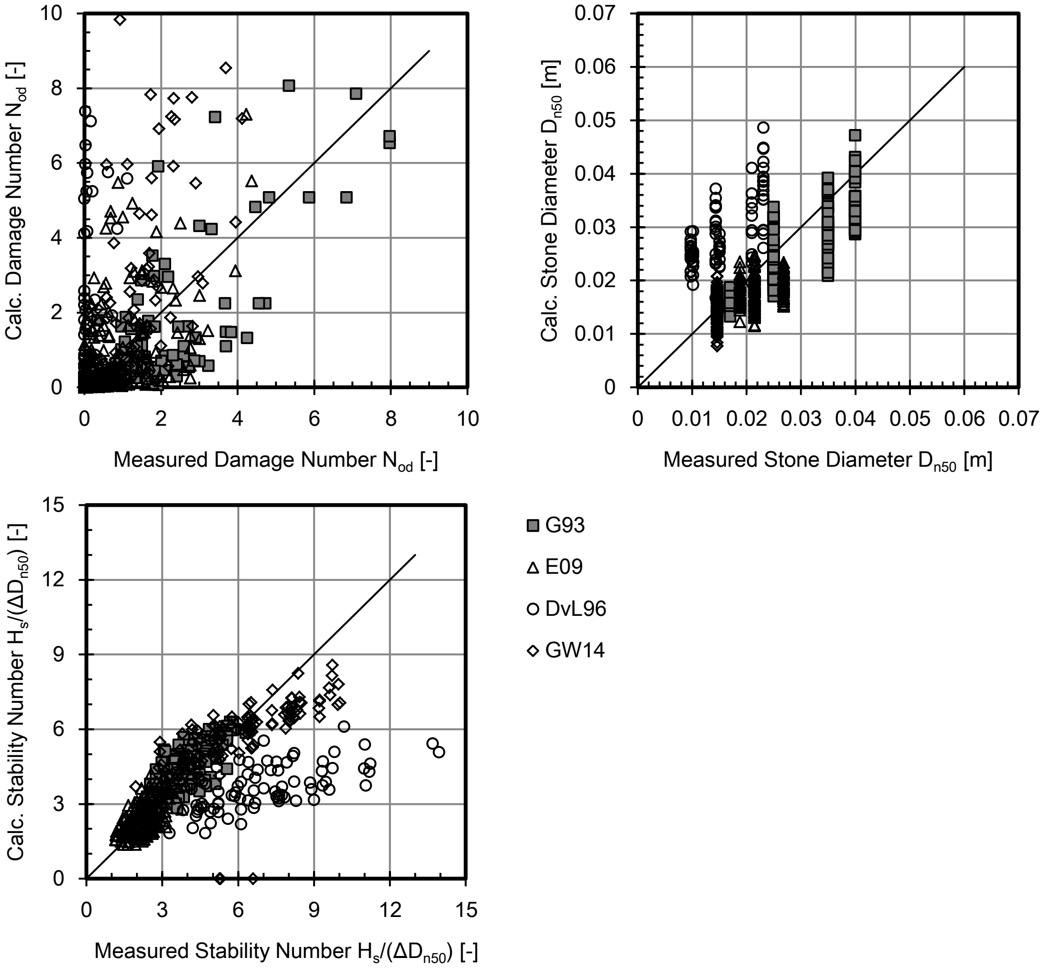

Figure 2: Predicted damage number (top, left), required nominal stone diameter (top right) and stability number (bottom) according to Eq. 2 (van der Meer, 1998) plotted against experimental results

Different from the two other formulae M13 is proposed as a design formula providing a somewhat conservative estimate of damage, required rock size and toe stability. Therefore the 90% upper confidence band (CU) was applied for the evaluation of M13. The CU of the stability numbers, $H_s/\Delta D_{n50}$ according to M13 (Fig. 3, bottom), of the required rock size, D_{n50} (Fig. 3, top right) and of the damage numbers, N_{od} (Fig. 3, top left) are given by factors 1.4, 1.3 and 1.6, respectively. These numbers refer to all tests; damage numbers have only been determined for tests with $N_{od} > 0.1$. These CU factors do not reflect significant deviations of predicted stability numbers and rock diameters from the model test results of Docters van Leeuwen (1996). The same applies for predicted damage numbers when compared with model test results of Doctors van Leeuwen and of van Gent and van der Werf.

Stability numbers, $H_s/\Delta D_{n50}$ according to GW14 (Fig. 4, bottom) have a 90% confidence interval (CI) that is defined by a factor 1.6 for all tests results and by a factor of 1.3 when considering only the test results of van Gent and van der Werf (2014). When calculating the required rock size, D_{n50} (Fig. 2, top right) the CI factor increases to 1.9 for all tests; a CI factor of 1.3 was found for the tests of van

Gent and van der Werf. Similar to vdM98 and M13 the prediction of damage numbers, N_{od} is vague (Fig. 2, top left). When only looking at the test results of van Gent and van der Werf the CI factor is 2.0 (for all tests with $N_{od} > 0.1$) and reduces to 1.5 for tests with $N_{od} > 0.5$. When considering all tests, there is little correlation between measured and predicted damage.

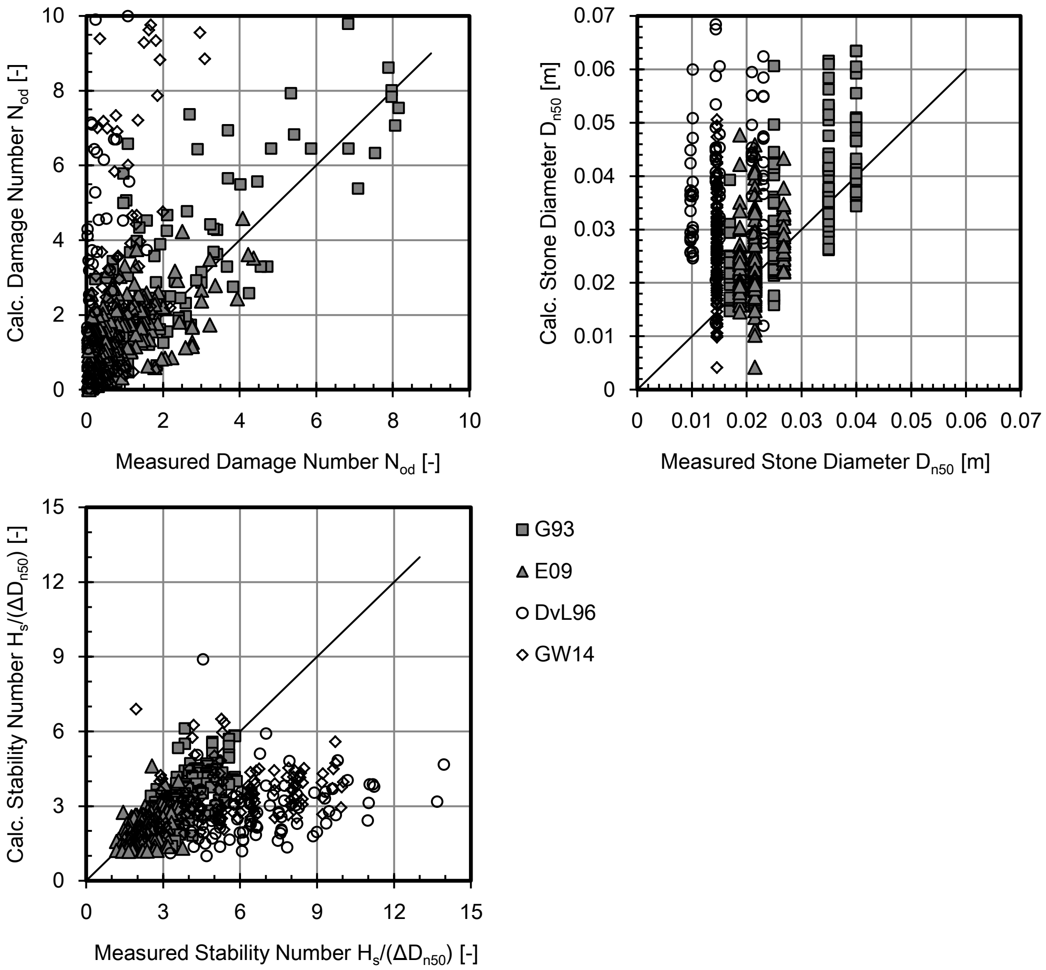

Figure 3: Predicted damage number (top, left), required nominal stone diameter (top right) and stability number (bottom) according to Eq. 3 (Muttray, 2013) plotted against experimental results

All three toe stability formulae (Eqs. 2, 3 and 4) provide a more or less rough estimate of the actual stability numbers and required rock diameters. The accuracy of all three formulae is significantly improved when they are applied to the data sets that had been used in the derivation of the respective formula. In this case the uncertainties of the predicted $H_s/(\Delta D_{n50})$ and D_{n50} are of order ±30%. These uncertainties increase to about ±60% when considering all model tests.

When toe stability formulae are used to predict damage, the results are of limited value, indicative at most. Even when the formulae are applied only to those test results that had been used in the derivation of the respective formula the uncertainties are close to ±100%. When considering results from all tests, extremely poor damage predictions are found.

It appears from the above that the most recent (M13 and GW14) and the most accepted (vdM98) toe stability formulae suffer from a lack of accuracy and general validity. Lack of accuracy refers to uncertainties of 30% to 60% in predicting stability numbers and rock diameters; damage number predictions bear even larger uncertainties. Lack of general validity refers to the finding that all three formulae perform significantly worse when applied to new data sets, despite the fact that parameters are well within the range of validity of the respective formula. In other words, the applicability of the empirical toe stability formulae to parameter combinations that deviate from those in the developer's own data sets is limited. This lack of general validity is considered as the main shortcoming of the three toe stability formulae (Eq. 2, 3 and 4). This has – to the authors' knowledge – not been noted earlier; the underlying reasons have to be investigated to clear the way towards a better toe stability formulae.

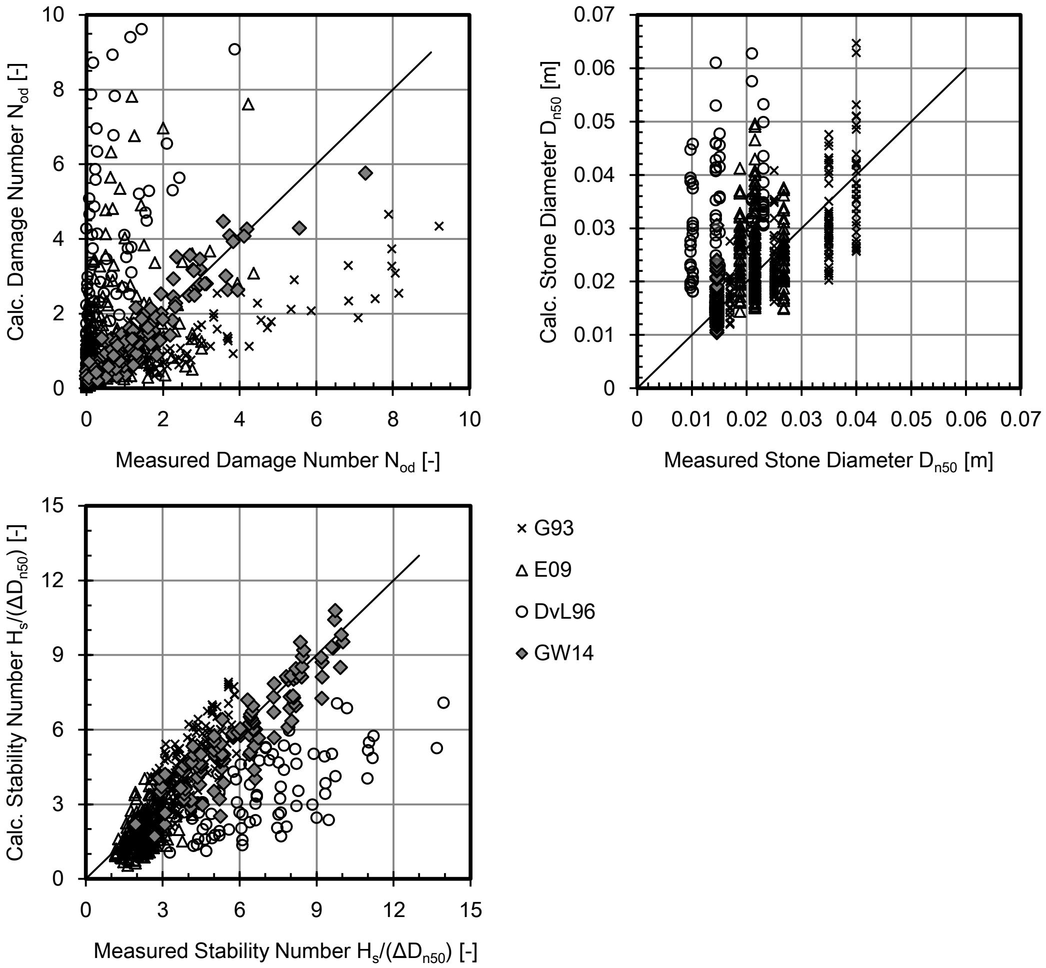

Figure 4: Predicted damage number (top, left), required nominal stone diameter (top right) and stability number (bottom) according to Eq. 4 (van Gent and van der Werf, 2014) plotted against experimental results

THE PROBLEM WITH DEPENDENT VARIABLES

The experimental studies that have been applied for this review are not fully consistent. The most obvious difference is the definition of damage, which may refer either to the accumulated number of displaced stones in a series of tests or to the number of displaced stones in a single test. On top of this a stone may be considered as displaced if it moved more than one stone diameter from its initial position or if it rolled away from the toe berm. Other differences between the experimental studies are for example the seabed slope, the breakwater slope or the presence of a bedding layer under the toe berm (see Table 1). Besides these reported differences between the toe berm experiments there are unassigned differences, the so-called model effects. The tests were performed by different experimenters in different labs, they used different control signals for the wave generator and they constructed and repaired the toe berms slightly different. All this may affect the outcome of the experiments and thus the agreement between predictive formulae and experimental results.

The disagreement that can be seen in Figs. 2, 3 and 4 is too large to expect it to just be model effects or the result of slight differences in the experimental studies. There seems to be a more fundamental problem with the existing toe stability formulae; they might for example miss out an important aspect of toe berm stability.

In order to confirm the appropriateness of the experimental data, the interdependencies of the basic dimensional parameters were analysed. As expected, most parameters show little or no correlation and thus can be considered as independent parameters. However two of the main parameters, the water depth above the toe, d_t and the wave height, H_s are not independent. The correlation of H_s and d_t is plotted in Fig. 5 (left) for tests with largely constant damage numbers ($0.5 < N_{od} < 1.5$) from all four model studies. The rock size and density, ΔD_{n50} is indicated by different symbols. The close relation of of H_s and d_t could be the result of depth limited wave conditions. This however is not the case; the relative wave height H_s/d (using here the water depth in front of the structure) of the same set of tests is plotted in Fig. 5 (right) and varies from 0.3 to 1.3 indicating that these tests cover a wide range of test conditions with and without depth limited waves.

An almost constant ratio of H_s and d_t was found, when the toe damage is close to $N_{od} = 1$. This ratio is largely independent of ΔD_{n50} (see Fig. 5, left). If we would draw conclusions on toe stability from this observation, we would arrive at ridiculous results. This is a direct consequence of the interdependencies of H_s and d_t in all four model studies, which has apparently nothing to do with toe stability. Both parameters, H_s and d_t are influenced by the experimenters' choices when defining the model dimensions and setting up the test programme. The close correlation between H_s and d_t might be the result of these choice. And if so, similar choices have apparently been made in the selection of test conditions in all four model studies.

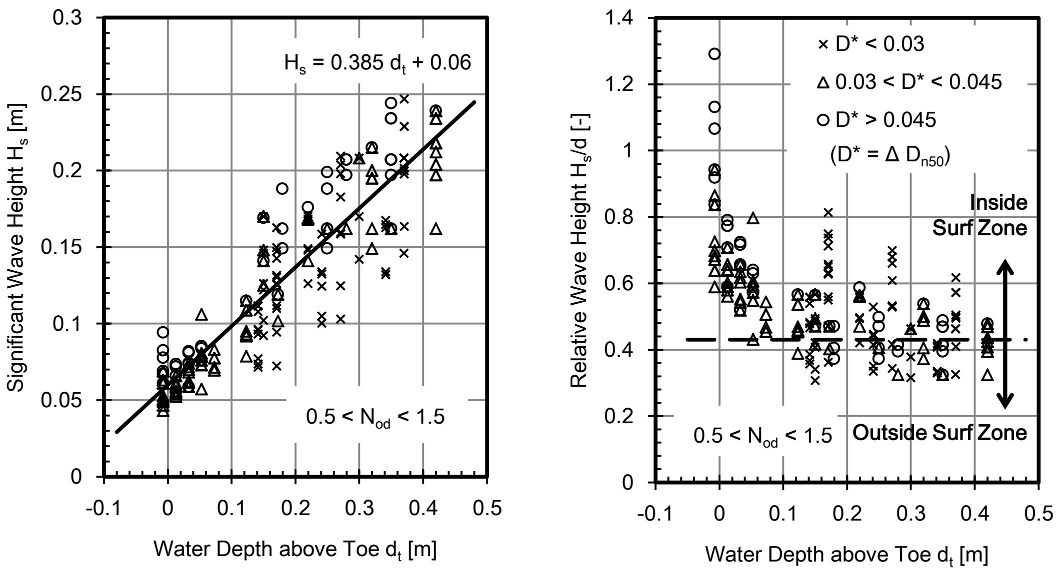

Figure 5: Relation of wave height and water depth on the toe berm: Interrelation in tests with moderate damage (left), occurrence of depth limited wave conditions in these tests (right)

Two of the main parameters for the toe berm stability, the wave height, H_s and the water depth above the toe berm d_t are not independent. This applies to all four model studies and might be the result of choices that have been made in setting up the test programmes. All attempts to improve the accuracy and predictive power of an empirical toe stability will inevitably make use of this close correlation between H_s and d_t. However, as this correlation has nothing to do with toe stability, it will result in a biased and thus not generally-applicable toe stability formula. This explains the severe shortcomings of Eq. 2 (vdM98), Eq. 3 (M13) and Eq. 4 (GW14) as illustrated in Figs. 2, 3 and 4. Most of the earlier toe stability formulae are likely to face the same problems as they are similar to Eq. 2 (see Muttray, 2013).

When performing wave flume experiments, the independence of test parameters should be routinely tested. However in practice the test parameters are combined in dimensionless numbers without checking for dependencies. There might be more empirical formulae in coastal engineering, which have been derived from wave flume tests and which are based on data sets with interdependent parameters. In consequence, these formulae may also be biased or may lack generality.

STEP-BY-STEP DEVELOPMENT OF A TOE STABILITY FORMULA

The wave height, H_s and the water depth above the toe berm, d_t are both relevant parameters for the toe berm stability. Excluding one of these parameters is thus not sensible and by the way, this would not necessarily overcome the problem of dependent parameters. A theoretical approach would be more promising; an empirical approach with physical meaningful parameter combinations might work as well. The latter route was selected for the step-wise development of an alternative toe berm stability formula. We will start with a simple case with a minimum number of influencing factors and proceed then with more complex cases taking into account an increasing number of influencing parameters.

The first case is a toe berm with zero submergence; hence the effect of d_t is excluded. The influence of the damage number, N_{od} is also excluded by selecting tests with damage numbers close to one (i.e. $0.5 < N_{od} < 1.5$). In this case two of the five most influential parameters can be neglected; the remaining three parameters are H_s, D_{n50} and Δ. This toe berm configuration is sketched in Fig. 6 (right), the observed stability numbers are plotted in Fig. 6 (left). If the toe berm is close to the water line, i.e. if d_t/H_s is close to zero, the stability numbers vary between 1.4 to 2.4, are in average about 1.8 and are largely constant. A toe berm stability formula of the following form would be the obvious choice in this situation:

$$\frac{H_s}{\Delta D_{n50}} \approx 1.8 \ ; \ (d_t = 0; N_{od} = 1) \tag{5}$$

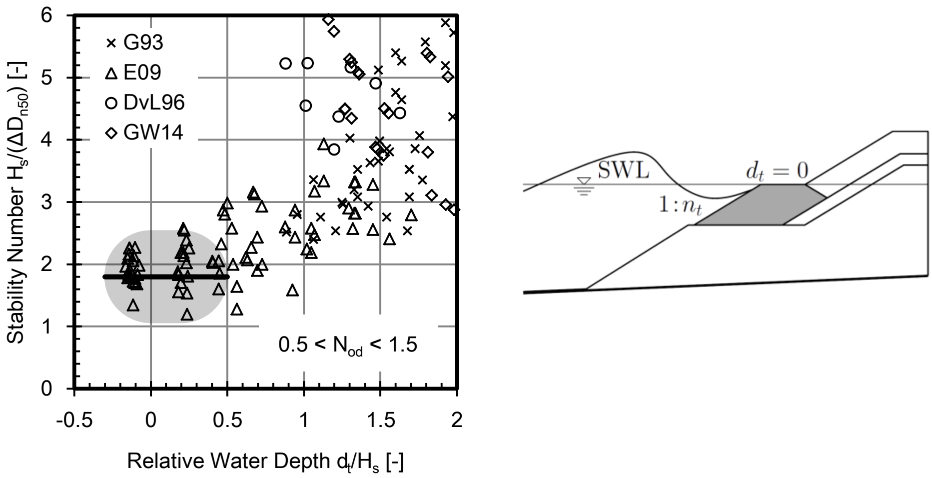

Figure 6: Stability of a toe berm with zero submergence (damage numbers close to one)

As expected the stability of a toe berm that extends to the water line is quite similar to the stability of the main armour. The Hudson formula predicts the same stability for the main armour as Eq. 5 for the toe berm when using the toe berm slope of the model tests (1:1.5) and a Hudson stability coefficient, K_d of 4:

$$\frac{H_s}{\Delta D_{n50}} = (K_d \ n_t)^{\frac{1}{3}} = (4 \cdot 1.5)^{\frac{1}{3}} = 1.8 \tag{6}$$

It may be arguable whether the Hudson formula refers to damage numbers, N_{od} of about 1. Nonetheless, the general agreement between Eqs. 5 and 6 provides some confidence in the validity of Eq. 5.

The next case refers to a submerged toe berm; the effect of d_t is thus included. This is a critical step as the interdependency of H_s and d_t may affect the outcome. In order to avoid further complicating factors the influence of the damage number, N_{od} is excluded as for the previous case. The following descriptive model (see also Fig. 7, right) has been applied:

- The wave induced water movements above the toe berm have an oscillating pattern;
- The flow velocities are related to the wave orbital motions even though the actual particle velocities may differ significantly from a progressive wave due to the wave reflection and due to the influence of the toe berm;
- Similar to wave orbital motions the motion amplitudes of the wave induced flow above the toe berm are decreasing over depth;
- This decay is in analogy to wave orbital motions related to the ratio of local water depth, d_t and peak wave length in front of the structure, $L_p = f(d, T_p)$.

The observed stability numbers are plotted in Fig. 7 (left) against d_t/L_p. In line with the above model the toe berm stability is increasing with increasing submergence and decreasing for long waves.

This variation is approximately linear; a toe berm stability formula of the following form may be considered:

$$\frac{H_s}{\Delta D_{n50}} = c_1 + c_2\,\frac{d_t}{L_p}\ \ ;\ \ (N_{od} = 1) \tag{7}$$

The experimental results are reasonably reproduced by a coefficients c_1 of about 1.8 (see also Eq. 5) and a coefficient c_2 of order 20 to 30. Eq. 7 is consistent with Eq. 5 and is in line with the above descriptive model. Therefore, Eq. 7 appears to be a plausible toe stability approach. Nonetheless it must be noted that H_s and d_t are numerators on both sides of the equation; this may affect the general validity of this stability formula. Further substantiation by additional model test results or by a sound theoretical analysis would be required.

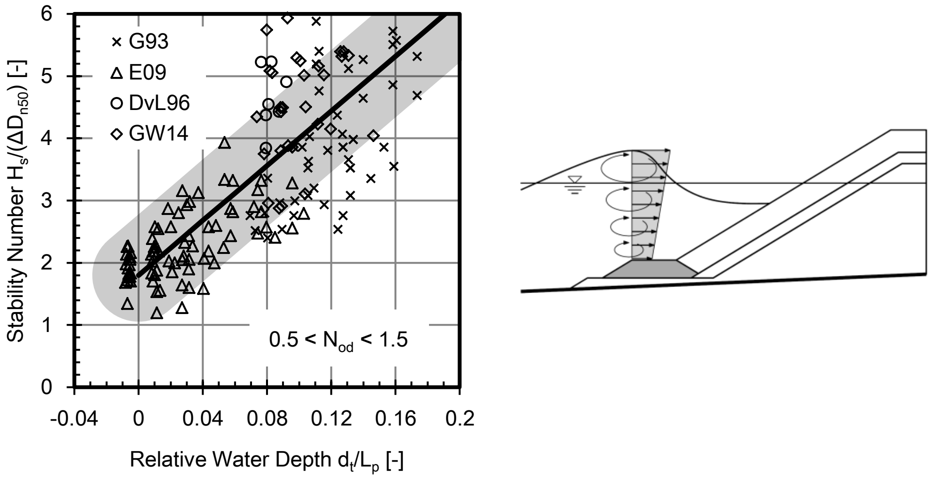

Figure 7: Stability of a submerged toe berm (damage numbers close to one)

When plotting measured stability numbers from model tests against d_t/L_p as done in Fig 7 (left) and in Fig. 8 (left) (the only difference between these plots is the scale on the y-axis) the scatter is increasing with increasing relative water depth. Different markers are used in Fig. 8 (left) for different seabed slopes. It can be seen that stability numbers on a gently sloping seabed (1:50) are larger than on a steep seabed (1:10). The scatter is thus at least partly related to the seabed slope.

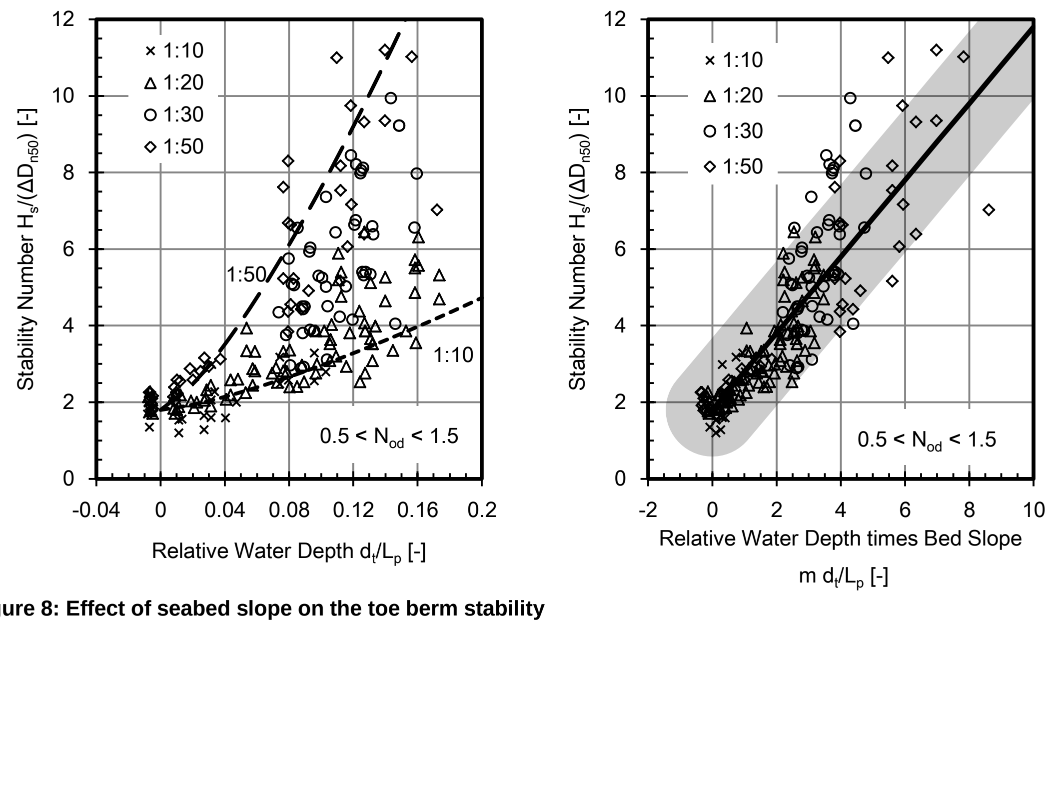

Figure 8: Effect of seabed slope on the toe berm stability

It appears further from Fig. 8 (left) that the minimum stability (if d_t/L_p is close to zero) is largely constant and independent of the seabed slope. The bed slopes becomes more relevant for larger values of d_t/L_p. When including the seabed slope, m in Eq. 7 it should be integrated in the second term on the right hand side and should not affect the first term. These requirements are met by a toe berm stability formula of the following form:

$$\frac{H_s}{\Delta D_{n50}} = c_1 + c_2 \frac{d_t}{L_p} m^{c_3} \;; \;\; (N_{od} = 1) \tag{8}$$

The experimental results (with $0.5 < N_{od} < 1.5$) are plotted in Fig. 8 (right) against $m\, d_t/L_p$. The solid line corresponds to coefficients c_1 of 1.8, c_2 of 1 and c_3 of 1. The overall trend of the measured toe stability is properly described by this line; the remaining scatter appears acceptable for this simplistic toe stability approach.

The effect of seabed slope seems to be significant; the seabed slope is apparently as important as the submergence of the toe berm or as the wave height. Stability numbers on a seabed with gradient 1:50 are about 2 to 4 times larger than on a 1:10 seabed slope. In other words, the weight of toe armour stones on a 1:10 seabed slope would have to be 10 to 60 times larger than on a 1:50 seabed slope. Although confirmed by the available data this appears suspicious from a practical point of view. The effect of seabed slope requires thus further substantiation by additional model tests or by a theoretical analysis. Against this background, Eq. 8 is proposed as a working hypothesis for the toe berm stability with a big question mark behind the actual influence of the seabed slope.

The final step in this analysis gives attention to the effect of damage number, N_{od}. The inclusion of other parameters in Eq. 8 is not advisable with respect to the not yet fully understood influence of m and d_t/L_p. The damage number will affect the stability of a toe berm regardless of the submergence. In other words, even if d_t is zero, the stability number should be increased when the damage increases. Therefore, the damage number should be a multiplier of the first and second term on the right hand side of Eq. 8. The following approach appears sensible for the effect of damage number on the toe berm stability:

$$\frac{H_s}{\Delta D_{n50}} = \left(c_1 + c_2 \frac{d_t}{L_p} m^{c_3} \right) N_{od}^{c_4} \tag{9}$$

The overall trend of the measured toe damage, N_{od} is reasonable reproduced with coefficients c_1 of 1.8, c_2 and c_3 of 1 and with c_4 of 1/3. With these coefficients the proposed toe berm stability formula (Eq. 9) simplifies to Eq. 10. The measured damage from all tests with $N_{od} > 0$ is plotted in Fig. 9 against the predicted damage (Eq. 10); the latter is a central estimate of the observed damage (solid line in Fig. 9, left and right). The upper bound of the observed damage (dashed line in Fig. 9, left) is about three times larger than the damage prediction by Eq. 10. The same data is plotted in Fig. 9 (right) with $N_{od}^{1/3}$ on the y-axis instead of N_{od}.

For toe berms that are close to the water line (i.e. with d_t close to zero) the second term in Eq. 10 disappears and the right hand side of Eq. 10 reduces to 1.8 $N_{od}^{1/3}$, which is the minimum toe berm stability. This minimum stability can be described by a Hudson type stability formula (see also Eq. 6); 1.8 $N_{od}^{1/3}$ corresponds then to $(K_d\, n_t)^{1/3}$. In other words, the minimum stability would be a function of Hudson stability coefficient, K_d and toe berm slope, n_t. The latter replaces the breakwater slope of the Hudson formula. A toe berm slope of 1:1.5 and a K_d coefficient of 4 are the equivalent of a damage number of $N_{od} = 1$ for a toe berm (see Eq. 6). A damage number of $N_{od} = 0.5$ might be more in line with the Hudson formula; the corresponding K_d coefficient is 2 for a toe berm with a 1:1.5 slope. The effect of toe berm slope can be incorporated in the toe stability formula by:

$$\frac{H_s}{\Delta D_{n50}} = \left(1.8 + \frac{d_t}{L_p} m \right) N_{od}^{\frac{1}{3}} = \left((4\, n_t)^{\frac{1}{3}} + \frac{d_t}{L_p} m \right) N_{od}^{\frac{1}{3}} \tag{10}$$

$$N_{od} = \max\{N_{od}, 0.25\} \;; \;\; d_t = \max\{d_t, 0\} \;; \;\; m = \min\{m, 50\} \;; \;\; n_t = 1.5$$

For practical applications Eq. 10 should not be used for damage level less than 0.25 (N_{od} should be set to 0.25 in this case), for seabed slopes less than 1:50 ($m = 50$ should be used for more gentle slopes) or for emerged toe berms (negative values of d_t should be replaced by 0).

The inclusion of the toe berm slope in Eq. 10 provides an approximate description of the increased stability when the toe berm flattens out. This effect has been observed in model tests; a proper description of this process would be useful for designing engineers. Nonetheless no tests with varying toe berm slopes have been performed in the four model studies; the validity of the Hudson analogy in

Eq. 10 could not be confirmed against model tests. Therefore results of Eq. 10 with toe berm slopes different from 1:1.5 should be considered as indicative. Further substantiation by model test or by a theoretical analysis will be required.

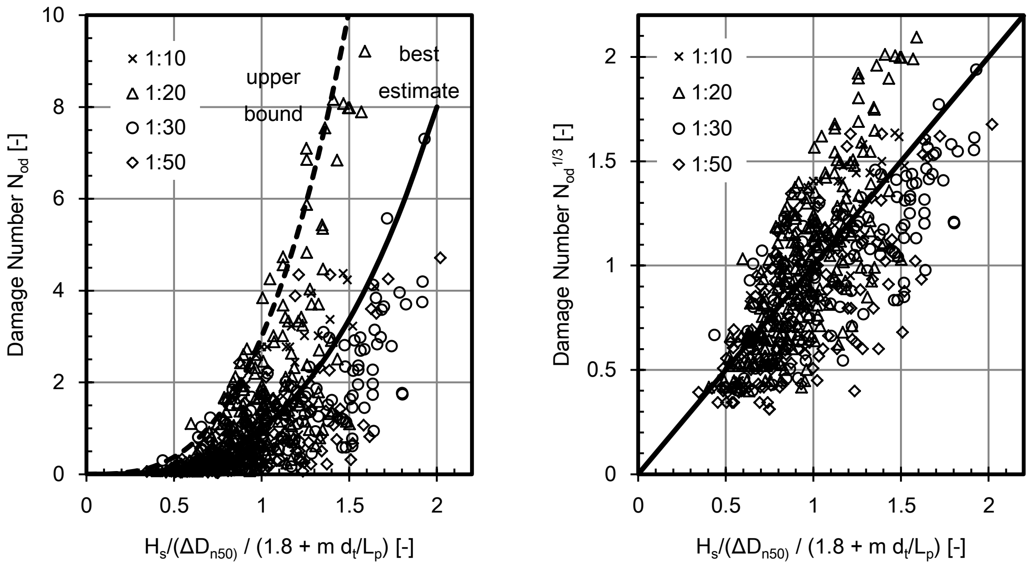

Figure 9: Progression of toe berm damage

The measured and calculated values of $H_s/(\Delta D_{n50})$, D_{n50} and N_{od} according to Eq. 10 are presented in Fig. 10 in the same format as vdM98, M13 and GW14 in Figs. 2, 3 and 4. When comparing the stability numbers, $H_s/\Delta D_{n50}$ according to Eq. 10 with model test results (Fig. 10, top right) the 90% confidence interval (CI) is defined by a factor 1.4. In other words, the observed stability is typically 1/1.4 to 1.4 times the predicted value. The CI of the required rock size, D_{n50} (Fig. 10, top left) is defined by a factor 1.5. When predicting damage numbers, N_{od} (Fig. 10, bottom) the CI factor increases to 3.0.

CONCLUSIONS AND RECOMMENDATIONS

Three widely used (vdM98) and recently developed toe stability formulae (M13 and GW14) have been validated against a comprehensive dataset with 687 model test results. The empirical formulae predict those model test results, which had been used for the derivation of the particular formula, reasonably well. The predictions are significantly less reliable when applied to other data sets. This lack of general validity is probably the result shortcomings in the underlying wave flume studies. In all data sets that have been considered in this study two main influence parameters for the toe berm stability, the wave height and the water depth above the toe berm, are not independent. Interdependence of parameters may easily lead to erroneous conclusions in empirical studies. This is the case for the three toe stability formulae that have been reviewed here and it is most probably also the case for many of the earlier toe stability formulae. It would be surprising if this issue would affect only toe stability; other empirical formulae that are based on wave flume experiments may be affected as well.

An alternative toe berm stability formula (Eq. 10) was developed by a step-wise approach starting with a simple case with a minimum number of influencing factors. More complex cases with increasing number of influence parameters were investigated subsequently. The following conclusions have been drawn:

- The toe berm stability, $H_s/(\Delta D_{n50})$ is the sum of two terms (see Eqs. 7 and 10).
- The first term refers to the minimum stability of a toe berm with virtually zero submergence and is nearly constant (i.e. is not significantly influenced by any of the parameters that have been varied in the tests).
- The second term describes the increased stability of a submerged toe berm and is thus proportional to d_t.

- The effect of submergence is probably a function of d_t/L_p. This hypothesis, although supported by the available data and by a simple descriptive model, cannot yet be confirmed due to the interdependence of d_t and H_s.
- The second term is further influenced by the seabed slope. The toe stability is reduced significantly on a steep seabed. However, the effect of seabed slope can only be determined without doubt when the uncertainties regarding the effect of submergence (previous item) have been clarified.
- The toe berm stability is likely to be increased for gentle toe berm slopes. The effect of slope gradient was included in the toe berm stability formula by a Hudson-style approach. The validity of this approach cannot be confirmed by the available data.
- The prediction of toe berm damage, N_{od} bears large uncertainties and is thus hardly advisable for practical applications.

Many aspects of the proposed toe stability formula (Eq. 10) have not yet been proved beyond doubt. Therefore this formula is far from being a design formula and should be considered as a working hypothesis. The new formula is believed to provide a more meaningful description of the toe berm stability than existing formulae. In view of the limitations of existing formulae, Eq. 10 is recommended as a benchmark for toe berm testing and design.

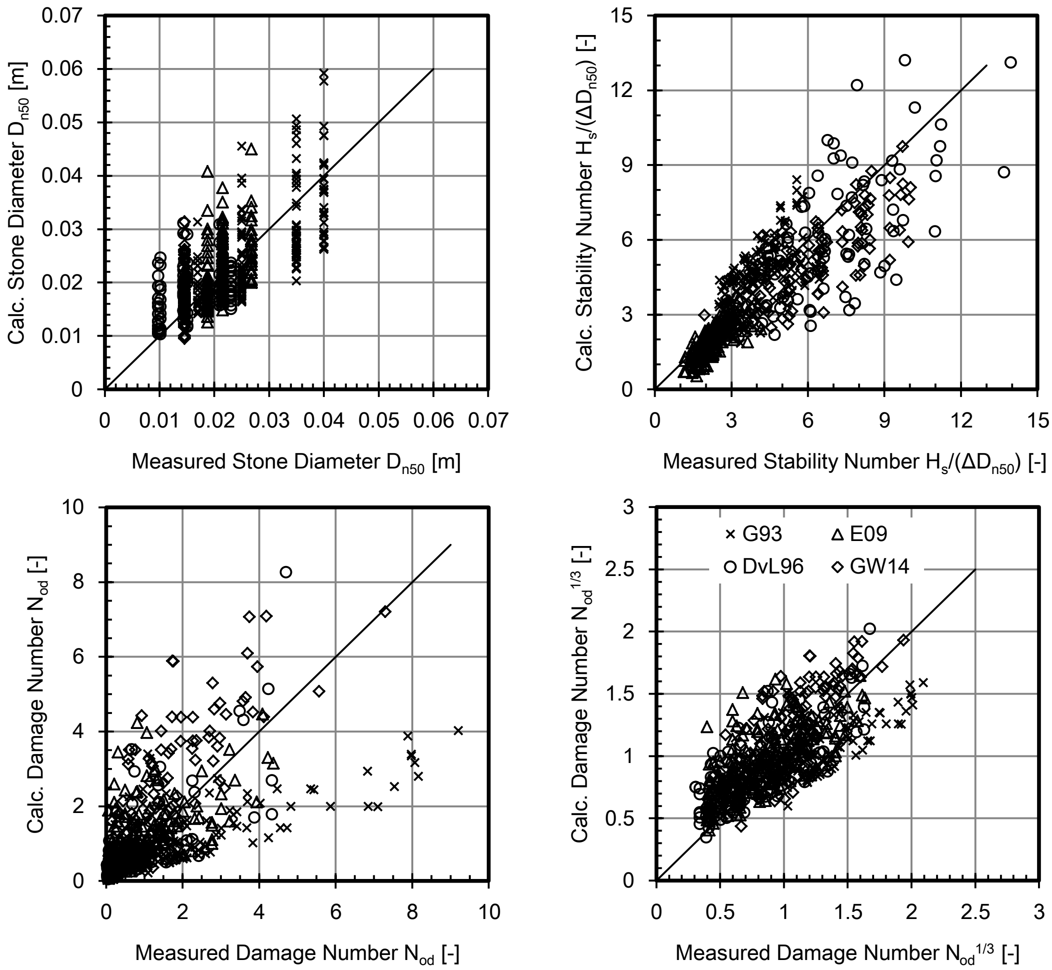

Figure 10: Predicted damage number (top, left), required nominal stone diameter (top right) and stability number (bottom) according to Eq. 10 plotted against experimental results

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